

AN INTRODUCTION TO CHROMATIC ABERRATION IN REFRACTORS

The popularity of high-quality refractors draws attention to color correction in such instruments. There are several points of confusion and misconceptions. I am not an optical designer -- just a used physicist who occasionally masquerades as an amateur telescope maker or amateur astronomer -- but perhaps I can clear things up a bit.

LONGITUDINAL CHROMATIC ABERRATION:

Nobody makes refractors with single-lens objectives because of longitudinal chromatic aberration, briefly called "longitudinal color". A simple lens has different focal lengths at different wavelengths. A well made lens will give a sharp image in any color you like, but that image will be blurred by the out-of-focus images of all other colors, combined. If you plot the focal length of a simple lens as a function of wavelength, across the visible spectrum, you will find that the difference between the minimum focal length (for blue) and the maximum (red) is about one and a half percent of the average focal length. Thus, a simple lens of nominal focal length 1000 mm might have a focal length in the red of 1007.5 mm and in the blue of 992.5 mm. Such a lens is said to have longitudinal chromatic aberration, across the visual, of one and a half percent, or 15 mm for a 1000 mm focal length.

Suppose you make such a lens, that is 100 mm diameter, and use it as an f/10 objective in a telescope. If you adjust an eyepiece to be in focus half way between red and blue, at the 1000 mm position, images of bright stars will be surrounded by an overlapping violet glow -- the out-of-focus red and blue images mixed -- that is 7.5 mm divided by 10, or 750 microns in diameter. The diameter of the Airy disc of a diffraction-limited f/10 objective is only about 13 microns, over 50 times smaller, so our simple-lens refractor won't work very well.

ACHROMATS:

Using two kinds of glass helps a lot, if the glass types are well chosen. With a positive lens of crown glass, and a negative one less strong, of flint glass, one can design a lens with longitudinal color reduced to about 0.05 percent of focal length. A lens that is a combination of two simple lenses is called a doublet. Such lenses are usually designed to focus blue and red light to the same position, with all other colors focusing at either slightly longer or slightly shorter focal lengths. A lens that brings two colors to a single focal point is

called an achromat. That 0.05 percent is the longitudinal distance between where red and blue focus, and where green focuses.

For such an achromat, the longitudinal chromatic aberration on our 100 mm f/10 objective is only 500 microns. If we focus for best sharpness in the green, star images will be surrounded by a purple haze that is only 50 microns in diameter -- four times the diameter of the Airy disc. The eye is most sensitive to green light, and some common glass combinations do better than I have described, hence a modern four-inch f/10 achromat made from conventional glasses is pretty good with respect to chromatic aberration. I have one -- a Vixen -- and though it does show a tad of purple haze, it is a fine instrument.

Whizzier optical materials make whizzier achromats. Use of first-generation "ED" glasses can reduce the longitudinal color by a factor of four, to 0.0125 percent of the focal length, and later ED glasses or fluorite can get another factor of two. A well-made four-inch f/10 doublet achromat using ED glass can have the purple blur the same size as the Airy disc, and a fluorite doublet of the same size can have the blur smaller than the Airy disc. That's why people like ED and fluorite doublets.

APOCHROMATS:

Still more pieces of glass can help more. A triplet objective -- using three pieces of glass -- can be designed to bring three colors of light to the same focal point: Remember that a garden variety achromatic doublet does so for just two. A lens system which brings three colors to the same focal point is called an apochromat. Three instead of two is not necessarily better, however -- it is a win only if the total length of the longitudinal blur, where all the other colors come to focus, is reduced.

There are even some wild glass types, with odd optical properties, whose use permits the design of a two-lens apochromat -- just two pieces of glass can bring three colors to the same focal point. Last I heard, however, these glasses were expensive, difficult to work, and not very durable. But perhaps we will see more of them in the future.

APOCHROMATS AND ACHROMATS COMPARED:

Some early triplet apochromats had as much longitudinal color as achromats of their day. For ease of photography, they were designed simultaneously to be in focus for visual use and for imaging with early,

blue-sensitive plates. Photography is still a consideration. Fluorite doublets have excellent color correction for visual work, but that correction often fails rapidly beyond the blue end of the spectrum, at wavelengths to which many photographic plates respond: If we could see just a little farther into the blue than we actually do, fluorite doublets would make lousy visual telescopes. But we don't, so they are superb: I've got one - a Vixen 90 mm f/9 -- and I love it.

In the visual wavelengths, the comparison is a little different. Most modern triplet apochromats have less longitudinal color than modern doublets that use conventional glasses, or early ED glasses, but good fluorite doublets generally equal the best of triplets, at least, across the visual. Triplets have other advantages, though, particularly for getting excellent images across a wide field, as useful for photography.

SCALING LONGITUDINAL COLOR:

How about for other sizes of telescopes? Well, things scale as follows: The linear diameter of the Airy disc depends only on the f number of the telescope, and on the wavelength. The actual formula is:

$$\text{Airy disc diameter} = 2.44 * \lambda * f,$$

where λ is the wavelength and f is the f-number. If you are puzzled because you thought bigger telescopes gave better resolution -- smaller diffraction patterns -- you are right, but so am I. This formula is for linear diameter, not angular diameter. An 8-inch f/10 telescope has twice the scale at the image plane, as a four-inch f/10, so that same-linear-size Airy disc takes up only half as much angle on the celestial sphere.

If you hold the focal length constant, and change the aperture of the lens, the diameter of the purple blur circle changes in proportion to aperture. But if you hold the aperture constant and change the focal length, the diameter of the purple blur circle remains constant.

So if you go from a four-inch f/10 to an eight-inch f/10, by simply scaling up the lens design by a factor of two, you get twice as big a blur (twice as much aperture), but you still have the same size Airy disc (still f/10), so the ratio of purple blur circle diameter to Airy disc diameter is doubled. If you want it back the way it was, you have to make your eight-inch telescope f/20 instead of f/10. That doesn't change the blur circle size, but it makes the diffraction disc twice as large. Now you see why large refractors often have long focal ratios.

Alternatively, if you speed up your four-inch $f/10$ to $f/5$, you haven't changed the blur circle size, but you have cut the diameter of the Airy disc in half, so all of a sudden that blur circle looks a lot bigger in proportion. Now you understand about all those colored star images in little rich-field refractors.

Unfortunately, there is more to color correction of refractors than longitudinal color. There is a major aberration we haven't considered.

SPHEROCHROMATISM:

Spherochromatism, or change in spherical aberration with wavelength, is a little confusing. To see what is going on, imagine we have set up a telescope for testing its figure -- by star-testing, by null-testing with a knife-edge, by a Ronchi screen, or whatever. Suppose further, that we have arranged to perform the test in various wavelengths of light -- perhaps by using a bunch of narrow-bandpass filters. The question is, is the figure equally good (or equally bad) at all wavelengths? The answer is, not necessarily, and a lens which has different figures at different wavelengths of light is said to have spherochromatism. Now clearly, correction for spherical aberration is a big deal in telescope objectives -- after all, that is the only thing that people who make paraboloids worry about. So it follows, that spherochromatism in a refractor matters a lot.

Our imaginary test setup provides a good way to understand the difference between longitudinal color and spherochromatism. A lens with no spherochromatism would give a perfect test result at every wavelength, but if it had longitudinal color, we would have to refocus the test apparatus, or change the knife-edge position, every time we changed filters. A lens with no longitudinal color would not require refocusing on wavelength change, but if it had spherochromatism, it would give different test results at different wavelengths.

What the designer of a simple visual objective will probably do is make the spherical aberration small at the wavelength to which the human eye is most sensitive -- green -- and trade off spherical aberration at other wavelengths till the correction is about the same in magnitude but opposite in sign in blue and red. It is hard to separate the effects of longitudinal color and spherochromatism under test, and indeed, what counts is their combination -- that's why designers publish spot diagrams in many colors. But for understanding, it is useful to distinguish them.

If a lens had no longitudinal color, then spherochromatism would be

visible in the star test, as color in the out-of-focus images of a star. Perhaps the edge of the out of focus diffraction pattern would look purplish, and the center greenish, or the other way around. I sometimes think I see this effect in my 98 mm f/6.7 Brandon refractor, whose objective is an one of Roland Christen's early triplet apochromats.

SCALING SPHEROCHROMATISM:

I don't have any generalizations about how spherochromatism varies with type of lens design. But its significance scales with the design, if you change both aperture and focal length in the same proportion. That is, if a four-inch f/10 focuses red light from the 70 percent zone half a wave longer than red rays from the center, then when you double all lens dimensions (making an 8-inch f/10), that half wave will double, too, and increase to a whole wave. Once again, large objectives are a difficult proposition.

LATERAL COLOR:

One chromatic aberration of eyepieces is worth mentioning, simply because it is common: In systems with decent objectives, it is often the most obvious chromatic aberration of the whole telescope. That is chromatic difference of magnification, or lateral color. It has nothing to do with color correction of the objective -- for the time being, we can assume that the objective is textbook perfect. The problem occurs when eyepiece focal length varies with wavelength. When that happens, since magnification is inversely proportional to eyepiece focal length, the magnification will be different at different wavelengths.

Suppose you are looking at the Moon. The image you see, perhaps, will be a small red Moon, a medium-sized green Moon, and a large blue Moon, all superimposed. (Red and blue might be reversed -- it depends on the eyepiece design.) The edge of the Moon will appear bluish -- that's the large blue image, sticking out from the superposition, and the boundaries of shadows and of materials of different brightnesses will have red and blue colored fringes.

Lots of binoculars have lateral color. You can see it in daylight as colored fringes at boundaries between light and dark areas, that are bigger at the edges of the field of view.

REFERENCES:

Most of the material I have described here is basic optics. It will be covered in most books on optics, and will be touched upon lightly in introductory college physics texts. A good source for more detail is

Harrie Rutten and Martin van Venrooij, 1988. *Telescope Optics*,
Willmann-Bell.